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A Novel Heuristic Approach for Distance- and Connectivity-based Multihop Node Localization in Wireless Sensor Networks

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Abstract The availability of accurate location information of constituent nodes becomes essential in many applications of wireless sensor networks. In this context, we focus on anchor-based networks where the position of some few nodes are assumed to be fixed and known a priori, whereas the location of all other nodes is to be estimated based on noisy pairwise distance measurements. This localization task embodies a non-convex optimization problem which gets even more involved by the fact that the network may not be uniquely localizable, especially when its connectivity is not sufficiently high. To efficiently tackle this problem, we present a novel soft computing approach based on a hybridization of the Harmony Search (HS) algorithm with a local search procedure that iteratively alleviates the aforementioned non-uniqueness of sparse network deployments. Furthermore, the areas in which sensor nodes can be located are limited by means of connectivity-based geometrical constraints. Extensive simulation results show that the proposed approach outperforms other previously published soft computing localization techniques in most of the simulated topologies. In particular, to assess the effectiveness of the technique, we compare its performance, in terms of Normalized Localization Error (NLE), to that of Simulated Annealing (SA)-based and Particle Swarm Optimization (PSO)-based techniques, as well as a naive implementation of a Genetic Algorithm (GA) incorporating the same local search procedure here proposed. Non-parametric hypothesis tests are also used so as to shed light on the statistical significance of the obtained results.

Keywords Wireless Sensor Networks · Node Localization · Flip Ambiguity · Harmony Search.

1 Introduction

The last decade has witnessed an evergrowing research interest in Wireless Sensor Networks (WSNs), which consist of hundreds or even thousand of nodes operating with high level of autonomy, while communicating to each other without the need of any wired link (Akyildiz et al., 2002). These densely-deployed sensor meshes permit to efficiently monitor a wide range of physical parameters in a cost-effective fashion. Originally restricted to military and defense applications, recent advances in wireless communications and electronics, along with the availability of low-cost smart sensors, have made WSNs also appealing for several emerging applications, such as infrastructure security, habitat monitoring (e.g. temperature, humidity, water, indoor air quality), precision agriculture, industrial sensing, traffic control, vehicle and animal tracking, etc.

In such applications, automatic and accurate location of the underlying sensor nodes is highly desirable in order to make collected data meaningful (Hu et al., 2004). Indeed, the knowledge of the location of the nodes plays an important role in the design of efficient network routing protocols and in security applications (Mauve et al., 2001). However, due to the constraints on the size, the cost and the limited energy available

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at sensor nodes, the installation of a Global Positioning System (GPS) on each device is not always feasible in practice, since it may jeopardize the network autonomy. Furthermore, GPS is not accessible in some environments, being generally not suitable for indoor and underground deployments. Consequently, most of the efforts so far have been aimed at developing alternative approaches to this problem, and thereby localization in WSNs is still deemed as an open research problem by the scientific community.

In this context, we focus on the anchor-based WSN scenario, where a few static nodes of the network (referred to as *anchor nodes*) know their exact positions in advance by means of either on-board GPS devices or their manual placement beforehand. The main goal is to estimate the coordinates of all non-anchor nodes, assuming that each sensor can infer the distance (subject to some error) to its neighbor nodes, based on Angle of Arrival (AoA) measurements (Niculescu et al., 2003), time-related measurements (e.g. Time of Arrival or Time Difference of Arrival (Savvides et al., 2001)) or Received Signal Strength Indication (RSSI) profiling techniques (Alippi et al., 2006). In particular, we focus on the latter, for which the most straightforward localization algorithm reduces to the statistical Maximum Likelihood (ML) estimation method. However, formalizing the localization problem as an ML estimation results in a multivariate non-convex optimization problem (Moré et al., 1997), for which different computationally-efficient approaches have been proposed in the literature.

Localization techniques can be broadly classified into *one-hop* and *multi-hop* localization schemes. In one-hop localization techniques, the non-anchor nodes to be localized must be located inside the coverage area (i.e. must be one-hop neighbors) of a minimum number of anchor nodes, while in multi-hop approaches this is not a necessary condition. In both cases, the localization algorithm exploits the distance and/or connectivity information – i.e., “who is in the range of whom” (Shang et al., 2004) – to estimate the positions of the whole set of non-anchor nodes in the network.

The use of connectivity information has coined the so-called *connectivity-based* and *range-free* localization concepts (see (Bulusu et al., 2000; Niculescu et al., 2001) and references therein). As for distance-based multi-hop localization algorithms, centralized and distributed approaches have been thoroughly reported in the related literature. In centralized localization algorithms such as those proposed in (Kannan et al., 2006; Biswas et al., 2004; Shang et al., 2003), each node only reports its estimated distances data to a fusion center, which takes the estimation task in charge, thus minimizing the computational load required at each node. On the contrary, in distributed schemes (He et al., 2003; Priyantha et al., 2003) each sensor node processes the locally available distance measurements to estimate its position, and eventually communicates with neighboring nodes to improve such estimation. Generally, centralized algorithms are less complicated, likely to provide more accurate location estimates but also less scalable, with respect to their distributed counterparts. Three main approaches for centralized localization algorithms can be found in the literature: Multidimensional Scaling (MDS) (Ji et al., 2002; Costa et al., 2006), Semi-Definite Programming (SDP) (Biswas et al., 2006) and stochastic optimization (Kannan et al., 2005, 2006). MDS consists of a set of data analysis techniques that represent the distance measurements in an N-dimensional space, based on which the relative coordinates of each node are obtained based on a starting distance matrix. On the other hand, semi-definite programming relaxes the original non-convex problem so as to obtain an approximate solution with reduced computational effort (Biswas et al., 2006; Tseng, 2007). Since the relaxation may incur significant estimation errors (Wang et al., 2008), a gradient search procedure (Liang et al., 2004) is often used to improve the initial solutions obtained by SDP (Biswas et al., 2004). Finally, the third class of techniques considers heuristic optimization methods for efficiently solving the localization problem, such as Simulated Annealing (SA) (Kannan et al., 2006), Particle Swarm Optimization (PSO) (Gopakumar et al., 2008) and Tabu Search (Shekofteh et al., 2010). In this paper we concentrate on a centralized distance-based multi-hop localization technique belonging to the third class of localization approaches.

Unfortunately, when the sparsity of the network is high enough to have a number of non-anchor nodes not connected to any anchor node, the network may become not uniquely localizable. In such situations, several different estimated topologies are compatible with the inter-node distance measurements, mainly due to the so-called *flip ambiguity* phenomenon. The flip ambiguity problem has been extensively analyzed in order to identify possible flipped nodes and mitigate their effects on the location estimations (Kannan et al., 2007, 2010). In particular, this effect can be catastrophic – from a localization point of view – when the estimation algorithm relies on the location estimations of *flipped* sensor nodes, because the localization error is propagated to subsequent estimations affecting, in turn, the estimation positions of the entire network. Figure 1 gives a glimpse of this concept: as the neighbors of node A (i.e.

nodes B, C, D, E) are nearly collinear, we have that

$$\begin{aligned} &d_{AB} \sim d_{A'B},\ d_{AC} \sim d_{A'C}, \\ &d_{AD} \sim d_{A'D},\ d_{AE} \sim d_{A'E}. \end{aligned} \tag{1}$$

It follows that node A can be reflected (*flipped*) with respect to the virtual line connecting its neighbors to position A', while satisfying the distance constraints and maintaining its connectivity with anchor nodes C and E.

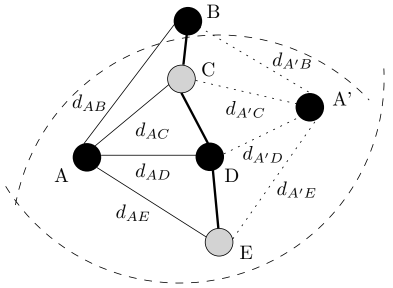


Fig. 1 Example of the flip ambiguity problem.

To alleviate this issue, an algorithm tackling the node localization problem in presence of the flip ambiguity phenomenon has been recently proposed in (Kannan et al., 2005). Basically, it consists of a two-phase optimization scheme relying on Simulated Annealing (SA) for both phases. In the first phase, SA is applied to obtain an initial estimate of the node locations by minimizing the squared error between the estimated and the measured inter-node distances. In the second stage, a refinement phase first identifies and then relocates the non-uniquely localizable nodes which may have been flipped during the first stage, by including an additional error term in the cost function, when the estimated location of a node violates the connectivity constraints defined by the network configuration. Similarly, Gopakumar and Jacob in (Gopakumar et al., 2008) have proposed to apply a Particle Swarm Optimization (PSO) algorithm to tackle the problem, but, unlike SA, they rely on a single execution of the PSO algorithm and, instead of minimizing the sum of squared errors between each non-anchor node and all its neighbors (anchor and non-anchor nodes), they only take into account those computed between each non-anchor node and its neighboring anchor nodes. Thus, in sparser scenarios, as the average node connectivity (and consequently the anchor to non-anchor connectivity) decreases, the single-hop PSO-based algorithm fails to obtain an accurate estimation of the positions of the whole non-anchor nodes set.

This work joins the upsurge of research on meta-heuristic centralized distance-based localization techniques. Specifically, we propose to combine the Harmony Search (HS) algorithm with a novel Local Search (LS) procedure that aims at mitigating the flip ambiguity phenomenon by exploiting the intrinsic connectivity constraints of the network configuration. In particular, the localization problem is formulated as the minimization of the sum of two different, yet mutually related terms: the first represents the squared error between the estimated and the measured inter-node distances, whereas the second establishes a penalty for all neighborhood violations in the estimated network topology. Based on this rationale, our proposal, hereafter referred to as HS-LS, can be regarded as a centralized connectivity- and distance-based localization approach with flipping mitigation. Extensive simulations run over 12 different network topologies will compare the performance of the proposed HS-LS with that of the aforementioned meta-heuristic schemes proposed in (Kannan et al., 2005; Gopakumar et al., 2008), as well as with that of a Genetic Algorithm (GA) incorporating the same local search procedure herein presented for a number of different topologies and connectivity ranges. Results will be discussed based on a number of statistics and hypothesis tests utilized for assessing their statistical significance.

This paper is organized as follows: in Section 2 the node localization problem is formally posed, whereas Section 3 delves into the proposed HS-LS algorithm. Section 4 thoroughly describes the alternative meta-heuristics (the algorithms in (Kannan et al., 2005; Gopakumar et al., 2008) and the implementation of a GA with the proposed LS procedure) against which the proposed approach is benchmarked. Next, Section 5 presents the simulation framework and discusses the obtained experimental results and finally, Section 6 concludes the paper.

2 Problem Statement

We consider WSNs composed by n nodes uniformly deployed in $T \triangleq [0,1] \times [0,1] \subset \mathbb{R}^2$, from which m nodes (with $m < n$) correspond to the anchor nodes whose coordinates $\mathbf{p}_i = (x_i, y_i) \in T$ ($i \in \{1, \ldots, m\}$) are perfectly known a priori. The remaining $n - m$ nodes are the non-anchor nodes, whose positions $\hat{\mathbf{p}}_i = (\hat{x}_i, \hat{y}_i)$, $\forall i \in \{m+1, \ldots, n\}$ are to be estimated by the localization algorithm. We define a $n \times n$ binary connectivity matrix $\mathbf{C}$, such that $c_{ij} = 1$ if sensor nodes i and j are within the connectivity range of each other *i.e.*, $r_{ij} \leq R$, where $r_{ij} \triangleq ||\mathbf{p}_i - \mathbf{p}_j||$ is the actual distance between nodes i and j ($\|\cdot\|$ denotes the Euclidean norm) and R represents the circular transmission range, common to all nodes. We further assume that each node knows which nodes it can communicate with, thus this information – embedded in matrix $\mathbf{C}$ – is *a priori* available. The measured inter-node distances d_{ij} can be obtained by resorting to any of the techniques introduced in Section 1, and will be modeled as

$$d_{ij} = \begin{cases} r_{ij} & \text{if } (i,j) \in \{1,\ldots,m\} \times \{1,\ldots,m\}, \\ r_{ij} + e_{ij} & \text{otherwise}, \end{cases} \tag{2}$$

where r_{ij} stands for the actual inter-node distance between node i and j, and e_{ij} represents the measurement error, modeled as a Gaussian distributed random variable with zero-mean and variance σ^2. Let us now define the set of neighbors of node i as

$$\mathcal{N}_i \triangleq \{j \in \{1,\ldots,n\},\, j \neq i : r_{ij} \leq R\}, \tag{3}$$

and its complementary set $\overline{\mathcal{N}}_i$, which contains the nodes located outside the connectivity range of node i. Note that the positions of the anchor nodes and the value of R determine the regions in which each non-anchor node may (or may not) be located. In particular, those non-anchor nodes inside the coverage area of a certain anchor node $i \in \{1,\ldots,m\}$ should be placed in the circle of radius R and centered in $\mathbf{p}_i = (x_i, y_i)$, whereas the remaining non-anchor nodes (i.e., those not connected to any anchor node) should be located outside the union of the circles of radius R and centered in all anchor nodes. Observe that this information, roughly depending on R and $\{\mathbf{p}_i\}_{i=1}^m$, can be exploited during the localization procedure to further refine the position estimates of the non-anchor nodes.

With these definitions in mind, the objective of our localization algorithm is to estimate the positions of all non-anchor nodes by minimizing the sum[1] of two objective functions, labeled as CF (Cost Function) and SCV (Soft Constraint Violation). CF simply represents the squared error between the estimated and the measured inter-node distances between nodes that are in the range of each other, and can be defined as

$$\text{CF} \triangleq \sum_{i=m+1}^{n} \left(\sum_{j \in \mathcal{N}_i} (d_{ij} - \hat{d}_{ij})^2 \right), \tag{4}$$

where d_{ij} and $\hat{d}_{ij} \triangleq \sqrt{(\hat{x}_i - \hat{x}_j)^2 + (\hat{y}_i - \hat{y}_j)^2}$ represent the measured and the estimated distances between node i and its neighbor j, respectively. SCV takes into account the connectivity neighborhood violations in each candidate topology, acting as follows: if a node j has been placed in the neighborhood of node i whilst $j \in \overline{\mathcal{N}}_i$ or, alternatively, its position is estimated such that $\hat{d}_{ij} > R$ while $j \in \mathcal{N}_i$, then it is likely the node has been incorrectly placed: in such situations, an error term $(\hat{d}_{ij} - R)^2$ is added to SCV[2]. Therefore, SCV can be formally defined as

$$\text{SCV} \triangleq \sum_{i=1}^{n} \left(\sum_{\substack{j \in \mathcal{N}_i \\ \hat{d}_{ij} > R}} (\hat{d}_{ij} - R)^2 + \sum_{\substack{j \in \overline{\mathcal{N}}_i \\ \hat{d}_{ij} \leq R}} (\hat{d}_{ij} - R)^2 \right). \tag{5}$$

The defined SCV metric helps alleviating the flip ambiguity phenomenon, especially in dense scenarios where a local minima in the CF metric may come along with some connectivity violations in the estimated topology. If so, an error term is added to the cost function SCV, hence increasing the overall cost.

Finally, we evaluate the goodness of the estimated topology by means of the *Normalized Localization Error* (NLE), which is calculated as

$$\text{NLE} \triangleq \frac{100}{R} \sqrt{\frac{1}{(n-m)} \sum_{i=m+1}^{n} ||\mathbf{p}_i - \hat{\mathbf{p}}_i||^2}. \qquad [\%] \tag{6}$$

It is important to emphasize that the computation of the above defined NLE parameter requires the knowledge of the real coordinates $\{\mathbf{p}_i\}_{i=m+1}^n$ of non-anchor nodes, thus it can not be regarded as an optimization metric, but instead serves as a measure of the accuracy of the estimated location solution $\{\hat{\mathbf{p}}_i\}_{i=m+1}^n$.

3 Proposed HS-LS Algorithm

To efficiently seek the optimum set of position estimates of all non-anchor nodes, we propose to hybridize the well-known heuristic HS algorithm with a novel local search procedure that attempts at reducing the flipping ambiguities in the candidate topology. As first presented by Geem, Kim, and Loganathan in (Geem et al., 2001), the HS algorithm belongs to the class of meta-heuristic population-based stochastic search approaches, and is based on mimicking the improvisation process of musicians when jointly composing a harmonious melody. This algorithm has been widely used in several hard optimization instances framed in distinct application fields, e.g. multicast routing (Forsati et al., 2008), engineering design (Liao, 2010), multiuser detection (Zhang et al., 2009; Gil-Lopez et al., 2009), or radio resource allocation (Del Ser et al., 2010, 2011). However, to the best of our knowledge, no previous work has

[1] Unity-valued weights and no normalization have been considered in the sum fitness, since the values of both constituent metrics result to be in the same order of magnitude and thus, comparable for the scenario at hand.

[2] Indeed, it is worth to notice that the proposed error term represents the minimum error due to a localization flip.

been reported in the scientific community dealing with the application of HS to the node localization problem.

Let us elaborate further on the roots of the HS algorithm, which in essence operates on a set of K candidate solutions or *melodies*, which are referred to as *Harmony Memory*. In our optimization framework, each melody encodes the position of all nodes of the network, thus the Harmony Memory can be denoted as $\{\{\hat{\mathbf{p}}_i^k\}_{i=1}^n\}_{k=1}^K$. The first m pairs of real numbers represent the actual (x, y) positions of anchor nodes (which, as said before, are assumed to be perfectly known in advance), whereas the remaining $n-m$ pairs correspond to the estimated coordinates of all non-anchor nodes of the network. Such K constituent melodies are iteratively refined – in terms of their associated sum metric $CF + SCV$ – by means of a stochastic *improvisation* process applied to every compounding element $\{\hat{x}_i^k, \hat{y}_i^k\}_{i=m+1}^n$ of the candidate solution. Observe that this stochastic improvisation procedure is only applied to the estimated positions of non-anchor nodes, which are further bounded by the topological constraints described in Section 2. We also impose these constraints in the initialization phase of the algorithm, where the starting candidate positions of the non-anchor nodes in the Harmony Memory are drawn at random from the areas defined by such topological constraints. After the improvisation procedure, the value of the sum metric function is computed for every improvised melody, based on which the best K melodies – out of the newly produced ones and those from the previous iteration – are kept for the next iteration. This refinement is repeated until a maximum number of iterations $\mathcal{I}$ is reached. In the following, we will describe the steps and the improvising operators used by our proposed HS-based localization algorithm.

Algorithm 1 Algorithmic description of the proposed HS-based localization approach.

```
1: for k=1 to K do
2:     pEstimated[k,1:m] ← ActualCoordinates[1:m]
3:     pEstimated[k,m+1:n] ← generateCoordinatesAtRandom(C)
4:     metric[k] ← evaluateFitness(pEstimated[k,1:n])
5: end for
6: for nIter=1 to $\mathcal{I}$ do
7:     for k=1 to K do
8:         p[k,1:n] ← applyHMCR(pEstimated[k,1:n])
9:         p[k,1:n] ← applyPAR(p[k,1:n], C)
10:        p[k,1:n] ← applyRSR(p[k,1:n], C)
11:        p[k,1:n] ← checkCoordinates(p[k,1:n],networkSize)
12:        if ($\mathcal{I} \equiv$ nIter (mod $\mathcal{I}_{LS}$)) then
13:            p[k,1:n] ← localSearchProcedure(p[k,1:n], C)
14:        end if
15:        metric[K+k] ← evaluateFitness(p[k,1:n])
16:    end for
17:    (metric[1:K], pEstimated[1:K,1:n]) ← filter(metric[1:2K],pEstimated[1:K,1:n],p[1:K,1:n])
18: end for
19: NLE ← calculateNLE(pEstimated[1,1:n], ActualCoordinates[1:n])
```

The proposed localization technique is sketched in Algorithm 1, in pseudocode notation. There, the connectivity radius R, the connectivity matrix $\mathbf{C}$ and the actual positions of the m anchor nodes are provided as input parameters to the algorithm. Moreover, $a \equiv b \pmod{c}$ denotes arithmetic congruence (i.e., a and b are congruent modulo c if the difference $(a-b)$ is a integer multiple of c), whereas a:b (with $a \leq b$ given integers) represents the sequence $\{a, a+1, a+2, \ldots, b-1, b\}$. First, the estimated positions of all nodes composing the Harmony Memory ($K \times n$-dimensional variable *pEstimated*) are initialized at random (within the topological constraints). Next, three different probabilistic operators are iteratively applied (lines 8 to 10) to *pEstimated* so as to produce tentatively refined candidate positions represented by the variable p, namely:

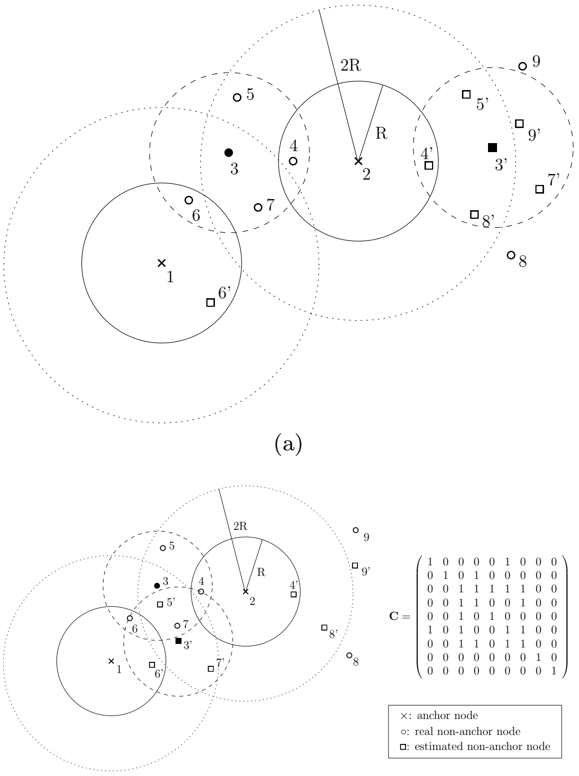
- The Harmony Memory Considering Rate, HMCR $\in [0,1]$, sets the probability that the new value for a certain note $(\hat{x}_i^k, \hat{y}_i^k)$ $(i \in \{m+1, \ldots, n\})$ is drawn uniformly from the values of the same note in all the other $K-1$ melodies in the Harmony Memory (HM).
- The Pitch Adjusting Rate, PAR $\in [0,1]$, establishes the probability that the new value for a given note $(\hat{x}_i^k, \hat{y}_i^k)$ (again, $i \in \{m+1, \ldots, n\}$) is randomly taken from its coverage area considering the geometrical constraints imposed by the anchor nodes for the non-anchor node at hand.
- The probability to pick a random value for the new note $(\hat{x}_i^k, \hat{y}_i^k)$ is controlled by another probabilistic parameter RSR (Random Selection Rate) $\in [0,1]$. As opposed to the PAR procedure, the RSR parameter operates network-wide along the subset $T_i \subset T = [0,1] \times [0,1]$, which is defined by the intersection of all geometrical constraints established by the connectivity range of the anchor nodes.

Once the operators have been applied to $\forall i \in \{m+1, \ldots, n\}$, the algorithm checks whether the notes of every newly improvised candidate coordinates of the Harmony Memory are within the network boundaries and eventually modifies such values to the closer boundary of T (line 11).

The proposed approach proceeds by performing a local search procedure every $\mathcal{I}_{LS}$ iterations. This procedure aims at improving the fitness value of the improvised candidate with potentially lowest metric value and is applied to each non-anchor node lying outside the connectivity range of any anchor node and whose any of its neighbors in the estimated topology differs from those imposed by the connectivity matrix **C**. Figure 2.a and 2.b depict an example of the application of the local search procedure to the estimated node $3'$ in a simplistic setup. In this scenario, the following notation is adopted: the anchor nodes are represented with crosses ($\times$), whereas the actual non-anchor nodes are marked as circles (● for the actual node 3, ○ for its actual neighbors 4, 5, 6 and 7) and the corresponding estimated coordinates as squares (■ for the estimated node $3'$, □ for its estimated neighbor nodes $4'$, $5'$, $6'$, $7'$, $8'$ and $9'$). Given that there are false neighbors in the estimated network topology violating the connectivity constraints imposed by **C** (i.e. $8'$ and $9'$), the proposed local search procedure is applied by sequentially executing the following steps:

1. Selection of a non-anchor node lying outside the connectivity range of any anchor node and whose neighbors in the estimated topology differs from those imposed by **C**, i.e. node $3'$.
2. Creation of the set of nodes that are going to be moved together with node $3'$: this group is filled with the non-anchor nodes that are not connected to any anchor node and are within the connectivity range of node 3 in the actual topology. In our case, since $c_{3j} = 1$ only for $j \in \{4, 5, 6, 7\}$, nodes 1 and 2 are anchor nodes and $c_{16} = c_{24} = 1$, this set is composed by nodes $5'$ and $7'$.
3. Identification of the anchor nodes located within the connectivity range of the actual neighbors of node 3; in our setup, nodes 1 and 2.
4. Move the node at hand (node $3'$) to the intersection of the annuli with inner and outer radii R and $2R$ respectively, centered in the selected anchor nodes 1 and 2, under the condition that the number of false neighbors decreases.
5. Place the actual neighbors, which are not connected to any anchor (i.e. nodes $5'$ and $7'$), randomly inside the circular coverage region centered in the new location of node $3'$ (see Figure 2.b).

The new generated candidate solutions are then evaluated (line 15) and the Harmony Memory is updated based on the global metric function CF+SCV (line 17). To this end, only those K harmonies improving the fitness with respect to those from the previous iteration are included in the next Harmony Memory. Once this has been done, the harmony memory is sorted in ascending order of the fitness values of its compounding melodies. Consequently, the potentially best candidate topology within a certain iteration will be given by $pEstimated[1, 1 : n]$. This procedure is repeated until a fixed number of iterations $\mathcal{I}$ is achieved and finally, the NLE value is computed in line 19 in order to assess the quality of the final estimate.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×: anchor node
○: real non-anchor node
□: estimated non-anchor node

(a)

(b)

Fig. 2 (a) Example of an scenario to which the local search procedure is applied; (b) resulting candidate topology after applying the local search procedure.

4 Related approaches

In this section we summarize different approaches presented in the literature for solving the node localization problem. Such schemes will be later used for assessing the performance of our proposed algorithm with respect to the state of the art in meta-heuristic localization in wireless sensor networks.

First, let us delve into the SA-based localization method presented in (Kannan et al., 2006). SA is essentially a stochastic optimization algorithm inspired by the physical process of annealing in metallurgy. As opposed to gradient-based search methods which employ the idea of steepest descent at each iteration, SA allows random uphill perturbations, thus preventing the search process from getting stuck in local minima by accepting worse candidate solutions based on probabilistic parameters. The specific SA localization approach in (Kannan et al., 2006) performs a two-stage optimization procedure: in the first phase, a preliminary estimate of the positions of non-anchor nodes is obtained by minimizing the CF objective function as defined in eq. (4). This minimization is accomplished by executing a first instance of the SA algorithm for a given number of iterations set beforehand. At the end of the first stage, the non-anchor nodes fulfilling all the connectivity constraints imposed by matrix $\mathbf{C}$ are identified and elevated to the status of *virtual* anchor nodes, whilst the remaining nodes (i.e., those non-anchor nodes undergoing the aforementioned flipping ambiguity) are relocated during the second *refinement* round of SA which minimizes a new cost function defined as

$$CF_{SA} \triangleq \sum_{i=m+1}^{n} \left(\sum_{j \in N_i} (\hat{d}_{ij}\text{-}d_{ij})^2 + \sum_{\substack{j \in \overline{N}_i \\ \hat{d}_{ij} \leq R}} (\hat{d}_{ij}\text{-}R)^2 \right). \quad (7)$$

The pseudocode of the SA-based algorithm is shown in Algorithm 2. First, the control temperature T_c is set at a high value to perform a highly explorative random search within the solution space of the problem. At each iteration, the control temperature T_c is decreased from T_0 to T_f according to line 26 (with $\alpha < 1$), whereas the distance gap ΔD is also set decreasing from its starting value ΔD_0 at a rate $\beta < 1$ (line 27). On the other hand, $N \cdot P \cdot Q$ randomly selected non-anchor nodes are perturbed (with $N \triangleq n - m$, and P and Q being arbitrary parameters). Each perturbed topology is then evaluated and accepted if it is characterized by a better fitness value with respect to the current one (lines 11-15). Otherwise, the solution with a worse fitness value is accepted with a probability $\exp\{\frac{-\Delta CF}{T_c}\}$ (lines 16 to 20), where ΔCF represents the difference between the current and previous values of the metric function. The control temperature T_c, which drives the acceptance rate of *worse* candidate estimates, *cools down* as the number of iterations increases.

On the other hand, the authors in (Gopakumar et al., 2008) proposed a PSO-based localization algorithm for WSNs. Unlike SA, PSO is inspired by the social behaviors and movement patterns of bird flocks or fish shoals. Each particle's movement is influenced by its best location estimate and the global estimate of the whole set of particles. Following the notation in (Gopakumar et al., 2008) and assuming a 2-dimensional localization scenario, let

$$\mathbf{pbest}_k \triangleq (pbest_k^x, pbest_k^y) \ (personal\ best)$$

denote the best position vector attained by the k-th particle during the search procedure, and let $\mathbf{gbest} \triangleq (gbest^x, gbest^y)$ represent the position of the global best particle in the K-dimensional particle swarm, i.e. the particle with the lowest metric function value. At the i-th iteration of the algorithm, the particles' velocities $\{\mathbf{v}_{k,i}\}_{k=1}^K \triangleq \{(v_{k,i}^x, v_{k,i}^y)\}_{k=1}^K$ and the estimated position vector $\{\mathbf{p}_{k,i}\}_{k=1}^K \triangleq \{(p_{k,i}^x, p_{k,i}^y)\}_{k=1}^K$ of all particles are updated according to

$$\begin{aligned} v_{k,i}^{\psi} = \omega v_{k,i-1}^{\psi} &+ c_1 r_1 (pbest_k^{\psi} - p_{k,i-1}^{\psi}) \\ &+ c_2 r_2 (gbest^{\psi} - p_{k,i-1}^{\psi}), \quad (8) \\ p_i^{\psi} &= p_{i-1}^{\psi} + v_i^{\psi}, \quad (9) \end{aligned}$$

where $\psi \in \{x, y\}$, r_1 and r_2 represents random numbers $\in [0, 1]$, w refers to the inertial weight and c_1 and c_2 are known as cognitive and social scaling parameters, respectively. The fitness function to be minimized by the proposed PSO algorithm is set to

$$CF_{PSO} \triangleq \sum_{j=1}^{N} \frac{1}{\mathcal{N}_j^{\Upsilon}} \sum_{i=1}^{\mathcal{N}_j^{\Upsilon}} (\sqrt{(x\text{-}x_i)^2 + (y\text{-}y_i)^2} - \hat{d}_i)^2, \quad (10)$$

where $\hat{d}_i$ corresponds to the noisy measured distance between the non-anchor node to be localized and its neighboring anchor nodes; N_j^{Υ} is the number of neighboring anchor nodes of node j; (x_i, y_i) are the coordinates of anchor nodes and (x, y) the coordinates of the target node to be estimated. It is important to note that the authors in (Gopakumar et al., 2008) explicitly impose that $\mathcal{N}_j^{\Upsilon} \geq 3 \ \forall j$, since no further mechanism is incorporated to the proposed PSO approach in order to account for possible flipping ambiguities. Nevertheless, we will use this *single-hop* algorithm in our benchmark so as to evince the importance of reducing the flip ambiguity phenomenon in sparse scenarios.

Finally, the meta-heuristics utilized for comparison in the next Section include a naive implementation of a population-based GA minimizing $CF + SCV$ by exploiting classical uniform crossover and uniform mutation as mating operators (with probability P_c and P_m, respectively), together with the same LS procedure described in Section 3.

5 Simulation Results

In order to assess the effectiveness of the proposed HS-LS algorithm when tackling the localization problem in WSNs, we have performed a number of computer simulations over synthetic networks with different levels of sparsity. In order to compare its performance against the previously mentioned soft-computing localization techniques, we have executed the PSO algorithm formulated in (Gopakumar et al., 2008) and the SA-based scheme proposed in (Kannan et al., 2006) over the same scenarios. Likewise, for the sake of completeness we also have included a naive implementation of a standard GA incorporating the local search procedure previously described (Section 3).

Algorithm 2 The SA approach proposed in (Kannan et al., 2006)

```
p[1:N] ← generateCoordinates(C)
$T_c \leftarrow T_0$
$\Delta D \leftarrow \Delta D_0$
$CF_{old} \leftarrow \infty$
while $T_c \geq T_f$ do
  for i ← 1 to $Q$ do
    indexNonAnchorPerm[1:N] ← randperm(1:N)
    for j ← 1 to $N$ do
      for k ← 1 to $P$ do
        p[indexNonAnchorPerm[j]] ← perturbNode(indexNonAnchorPerm[j], $\Delta D$)
        $CF_{new}$ ← evaluateFitness(p[indexNonAnchorPerm[j]])
        $\Delta CF \leftarrow CF_{new} - CF_{old}$
        if $\Delta CF \leq 0$ then
          acceptPerturbedNode(p[indexNonAnchorPerm[j]])
          $CF_{old} \leftarrow CF_{new}$
        else
          randomvalue ← uniformRandomValue(0,1)
          if randomvalue ≤ exp(-$\Delta CF/T_c$) then
            acceptPerturbedNode(p[indexNonAnchorPerm[j]])
            $CF_{old} \leftarrow CF_{new}$
          end if
        end if
      end for
    end for
  end for
  $T_c \leftarrow \alpha \cdot T_c$
  $\Delta D \leftarrow \beta \cdot \Delta D$
end while
```

The simulation framework consists of 12 different network topologies generated by uniformly placing $n = 200$ nodes in $T \triangleq [0,1] \times [0,1]$. In all such topologies, $m = 20$ nodes are set as *anchor* nodes, hence their positions are assumed to be known *a-priori* and fed to the algorithms. Moreover, we have varied the connectivity radius $R \in \{0.13, 0.15, 0.17\}$, so as to model 3 different network sparsity levels, each composed by 4 topologies. In particular, $TOP1$ to $TOP4$ represent the sparse topologies class ($R = 0.13$); $TOP5$ to $TOP8$ constitute the class of medium-sparse topologies ($R = 0.15$); and $TOP9$ to $TOP12$ form the class of dense topologies ($R = 0.17$). Finally, the inter-sensor distance measurements (2) are assumed to be based on RSSI, which is commonly affected by log-normal shadowing with standard deviation of the errors proportional to the actual distance r_{ij} between nodes i and j (Liu, 1998). Without loss of generality, in the following and for all the scenarios, the measurement errors e_{ij} are considered constant through all experiments for a given topology, with values drawn from a Gaussian distribution with zero mean and variance given by $\sigma^2 = \lambda^2 \cdot r_{ij}^2$, with $\lambda = 0.1$.

Table 1 summarizes the parameters setup employed by the different algorithms and deriving from a preliminary simulation campaign conducted to choose the most effective configurations. For the sake of the brevity, this preliminary analysis is omitted.

PSO	SA	GA-LS	HS-LS
$w:\ [0.8, 0.7]$ $c_1:\ [0.8, 0.6]$ $c_2:\ [0.8, 0.6]$ $K:\ 100$ $\mathcal{I}:\ 2000$	$T_{c,i}:\ 0.1$ $T_{c,f}:\ 10^{-11}$ $P:\ 10$ $Q:\ 2$ $\alpha:\ 0.80$ $\beta:\ 0.94$ $\Delta D_0:\ 0.1$	$P_c:\ 0.9$ $P_m:\ 0.01$ $K:\ 50$ $\mathcal{I}:\ 2000$	HMCR: 0.9 PAR: 0.01 RSR: 0.01 $K:\ 50$ $\mathcal{I}:\ 2000$

Table 1 Parameters setup used for the PSO, SA, GA-LS and HS-LS algorithms.

First, with the goal of analyzing the computational complexity, it is worth to characterize each approach in terms of required number of fitness evaluations. On the one hand, HS-LS and GA-LS employs a fixed number $\mathcal{I} = 2000$ of iterations while, at each iteration, the objective function is evaluated $K = 50$ times (one for each newly generated candidate solution). Therefore, in each trial the overall number of fitness evaluations for both the algorithms is equal to $K \cdot \mathcal{I} = 10^5$. Moreover, in these algorithms the local search procedure LS is applied to the best candidate topology every $\mathcal{I}_{LS} = 100$ iterations. Regarding the PSO scheme (Gopakumar et al., 2008), a swarm size of $K = 100$ particles is evaluated during $\mathcal{I} = 2000$ iterations. It follows that, in each trial, PSO performs $K \cdot \mathcal{I} = 2 \cdot 10^5$ fitness evaluations. Finally, the number of fitness evaluations performed by SA (Kannan et al., 2006) at each value of

#	R	Value	PSO in (Gopakumar et al., 2008)	SA in (Kannan et al., 2006)	Naive GA-LS	Proposed HS-LS
1	0.13	Mean	246.02	85.08	38.17	34.37 (◇)
		Min	222.06	45.06	22.73	17.22 (◇)
		Std	11.60	17.47	10.29	13.69
2	0.13	Mean	278.55	43.44 (◇)	64.29	57.52
		Min	245.73	16.31 (◇)	33.90	30.72
		Std	11.99	15.92	24.67	22.53
3	0.13	Mean	251.46	34.19 (◇)	54.98	54.68
		Min	217.48	13.01 (◇)	27.68	29.33
		Std	23.04	18.10	16.08	12.30
4	0.13	Mean	274.20	25.91 (◇)	35.91	34.75
		Min	211.13	9.17 (◇)	24.15	21.29
		Std	34.79	8.28	9.28	9.09
5	0.15	Mean	205.41	79.80	24.83	17.79 (◇)
		Min	183.14	17.48	16.41	10.69 (◇)
		Std	12.78	30.62	7.57	6.58
6	0.15	Mean	197.28	28.42	19.64	16.75 (◇)
		Min	180.86	7.96 (◇)	16.18	11.18
		Std	11.70	11.95	6.03	5.94
7	0.15	Mean	211.12	26.88	19.60	16.91 (◇)
		Min	176.31	7.63 (◇)	15.27	10.31
		Std	16.64	17.49	2.76	6.24
8	0.15	Mean	187.50	39.19	21.47	19.56 (◇)
		Min	171.54	8.57 (◇)	15.58	14.46
		Std	8.59	22.49	2.72	2.42
9	0.17	Mean	173.43	43.11	15.21	12.17 (◇)
		Min	156.39	5.40 (◇)	10.77	7.92
		Std	10.27	32.98	3.17	3.89
10	0.17	Mean	185.67	36.53	18.22	16.61 (◇)
		Min	172.56	4.95 (◇)	11.88	10.85
		Std	8.56	28.60	4.03	4.15
11	0.17	Mean	156.34	37.55	15.36	13.30 (◇)
		Min	140.86	5.22 (◇)	12.90	10.76
		Std	9.58	23.16	1.69	1.83
12	0.17	Mean	153.40	25.08	12.90	9.89 (◇)
		Min	137.97	5.59 (◇)	10.88	7.61
		Std	8.04	14.92	1.65	2.56

Table 2 NLE statistics obtained by the SA in (Kannan et al., 2006), the PSO in (Gopakumar et al., 2008), a naive implementation of a GA algorithm with the proposed local search (LS) and the proposed HS-LS. Best values of the NLE mean and min statistics among all compared algorithms have been highlighted with (◇).

the control temperature, during the first optimization phase is equal to $(n - m) \cdot P \cdot Q$. Unfortunately, the number of fitness evaluations performed during the refinement phase cannot be determined in advance, as the number of non-anchor nodes promoted to virtual anchor nodes is variable. However, we have verified that SA computes, on average, around $7.1 \cdot 10^5$ fitness evaluations during each trial. Thus, HS-LS and GA-LS reduce the computational load with respect to the PSO and SA counterparts in approximately 2:1 and 7:1 ratios, respectively. We remark that the rationale of selecting configurations with different complexity levels lies on the aforementioned preliminary off-line campaign, during which we could verify that, by using the parameters setup resumed in Table 1, the simulation results of each algorithm become stationary and/or comparable (in terms of the same order of magnitude in the results).

Table 2 shows the Monte Carlo results for the 12 simulated scenarios. Since all the tested methods are stochastic, 30 independent runs of each algorithm over a given scenario have been performed. Consequently, the table includes, for each algorithm and scenario, the mean, the minimum value and the standard deviation of the NLE after $\mathcal{I}$ iterations. It is important to notice that, independently on the particular fitness function employed by the different stochastic algorithms to explore the solution space, the NLE indicator (6) enables a fair comparison among the approaches. Indeed, it represents the deviation of an estimation of the sensor nodes' locations with respect to the real topology, normalized by the connectivity radius. Thus, assuming that the estimate is unbiased, the NLE can be interpreted as the ratio of the standard deviation to the connectivity radius. As aforementioned in Section 2 note

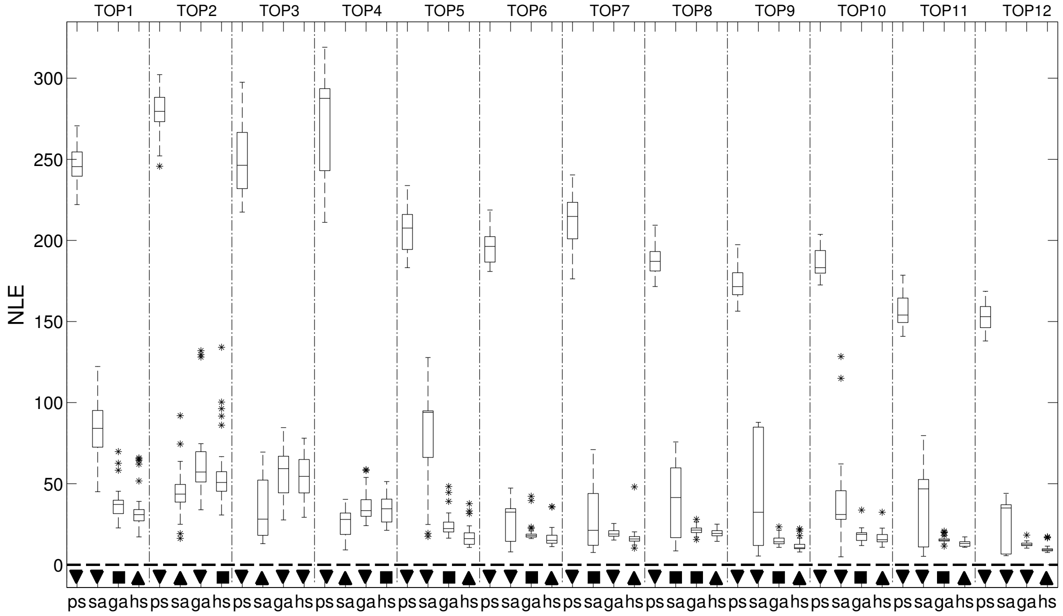

Fig. 3 Boxplot of the NLE values of the different algorithms over the 12 network scenarios. ps, sa, ga and hs denote, respectively, the PSO-based, the SA-based, the GA-LS and HS-LS approaches. Moreover, triangles (▲), reversed triangles (▼) and squares (■) are used to mark in each group, the best median and the difference with respect to the best median with or without statistical significance, respectively.

that, being the original topology unknown, the NLE cannot be directly employed as fitness function during the search phase, while it can be employed as an *a-posteriori*, yet objective, quality assessment indicator. First observe that the mean and the standard deviation of the NLE obtained by the HS-LS localization approach are in general lower than those achieved by the SA and the PSO algorithms, and similar (but still better than) to those obtained by the GA-LS scheme. On the other hand, the best (minimum) NLE values are in general lower for the SA – though it needs 7 times more function evaluations than its GA-LS and HS-LS counterparts –, similar for the GA-LS and HS-LS schemes, but significantly higher for the PSO technique.

Figure 3 shows the distributions of the NLE values obtained by all the algorithms over the 12 topologies: there, whiskers are used to represent the lowest and largest NLE values of the corresponding distribution, the boxes delimit the lower and the upper quartiles of the distributions, the medians are depicted with solid lines and the observations that may be considered outliers are possibly marked with asterisks. In the figure, ps, sa, ga and hs denote, respectively, the PSO-based, the SA-based, the GA-LS and HS-LS approaches. At a glance it can be observed that the PSO approach is characterized by the highest NLE distributions in all the experiments. As previously stated, the bad behaviour of this technique is to be imputed to the excessive sparsity of the scenarios, not balanced by any countermeasure to alleviate the catastrophic affects of the flip ambiguity phenomenon. Indeed, we have verified that, even increasing up to seven times the number of fitness evaluations, the PSO technique is unable to accurately solve the localization problem for the given scenarios. Regarding the SA-based approach, it can be observed that it is quite effective on the class of sparse topologies, especially on $TOP2$, $TOP3$ and $TOP4$ where it could obtain the best median values of the NLEs. As the average density of the networks increases, the distributions of the NLE values obtained by SA are more dispersed and characterized by higher medians with respect to GA-LS and HS-LS counterparts. Finally, GA-LS and HS-LS reveal the most stable behavior, macroscopically highlighted by equally-balanced lower and upper quartiles, whiskers close to the quartile boundaries and a relatively low number of outliers.

In order to statistically compare the distributions of the NLE values obtained by the different algorithms on each scenario (a group), we performed a non-parametric test, namely the Kruskal-Wallis test. The latter represents the non-parametric version of the classical oneway ANOVA, and is an extension of the Wilcoxon rank sum test to groups larger than 2. Briefly, the test compares the medians of the group, and returns the p-value for the null hypothesis that all samples are drawn from the same population (or equivalently, from different populations with the same distribution) (Hollander et al., 1987). If the p–value is lower than α, we can deduce that the null hypothesis does not hold, that is, at least one sample median in the group is significantly different from the others, with $(1-\alpha)$ percent level of confidence. Then, to determine which sample medians are statistically different, we have applied the multiple comparison procedure with $\alpha = 0.05$ (thus, with a 95% level of confidence) (Hochberg et al., 1987). The results of such procedure are depicted in Figure 3, by means of triangles (▲), reversed triangles (▼) and squares (■). In detail, within each group, a triangle marks the distribution with the best median (i.e., the lowest), while a reversed triangle and square mean, respectively, that the median of the corresponding distribution is larger than the best median of the group with or without statistical significance. We can observe in this plot that HS-LS produces the best NLEs results over 9 scenarios (all except $TOP2$, $TOP3$ and $TOP4$). In the remaining scenarios, SA achieves the best results, but with statistical significance with respect to HS-LS only in one scenario ($TOP3$). Finally, GA-LS, though quite stable and effective, could never obtain the best median, while its worse results with respect to the best median distribution have a statistical significance in 5 scenarios ($TOP2$, $TOP3$, $TOP4$, $TOP7$ and $TOP12$).

6 Concluding Remarks

In this paper we have presented a novel meta-heuristic localization technique for wireless sensor networks based on the harmony search algorithm, which is further aided by a local search procedure aiming at alleviating the so-called flip ambiguity phenomenon. The proposed algorithm exploits the information on the node connectivity by imposing geometrical constraints in order to restrain the areas where sensor nodes can be placed. Through extensive computer simulations, we have shown that our approach embodies a cost-effective centralized localization scheme outperforming, for most of the simulated scenarios, other recently proposed meta-heuristic strategies such as SA, PSO and a naive GA incorporating the local search procedure here presented.

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