

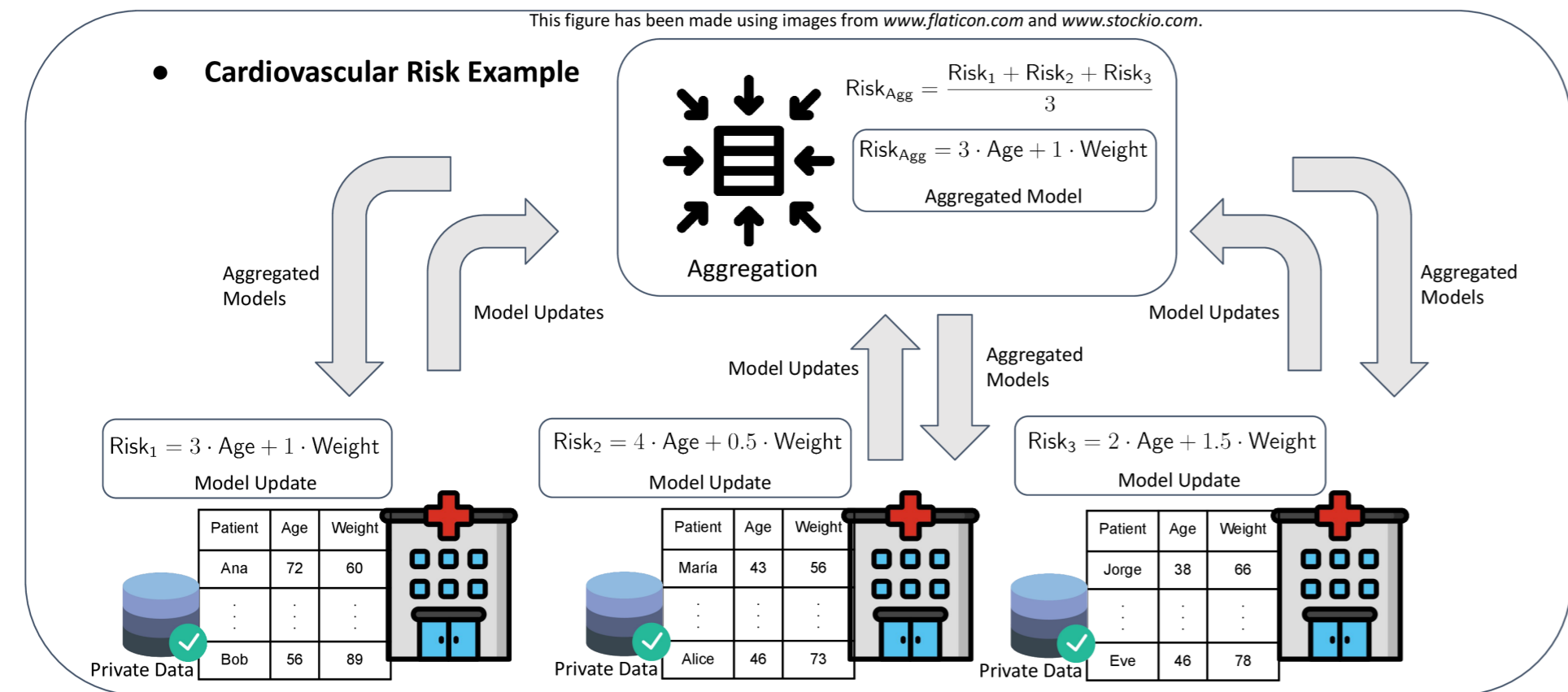
# Practical Multi-Key Homomorphic Encryption for Efficient Secure Federated Average Aggregation

A. Pedrouzo-Ulloa<sup>1</sup> A. Boudguiga<sup>2</sup> O. Chakraborty<sup>2</sup> R. Sirdey<sup>2</sup> O. Stan<sup>2</sup> M. Zuber<sup>2</sup>  
<sup>1</sup>apedrouzo@gts.uvigo.es, <sup>2</sup>name.surname@cea.fr

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## Optimizing HE for Federated Average Aggregation

**Federated Learning:** Many works address the problem of secure aggregation in FL [1]. However, to the best of our knowledge, HE has not been yet fully optimized for this setting.

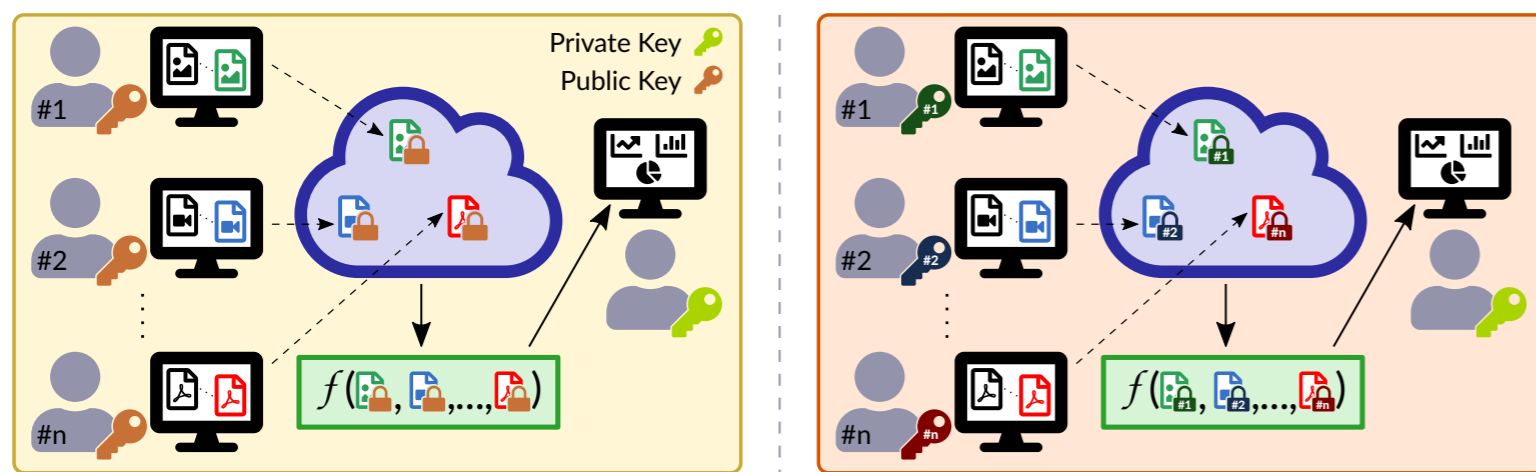


**Main objective:** Tailor and optimize HE constructions for secure average aggregation.  
**Main contribution:** A lightweight communication-efficient multi-key approach suitable for the Federated Averaging rule [2].

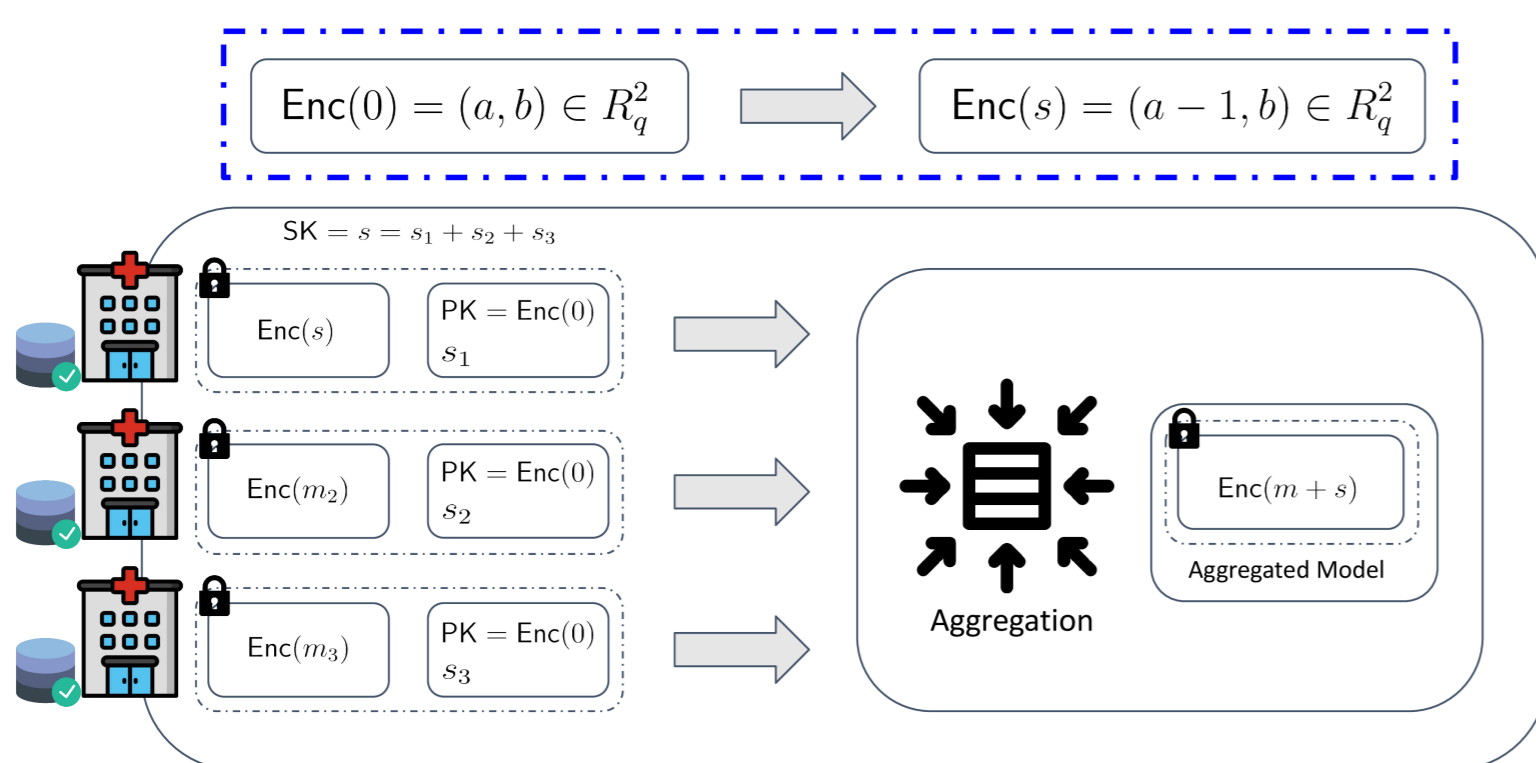
- Communication cost per party is reduced approximately (1) by a half with RLWE, and (2) from quadratic to linear in terms of lattice dimension if considering LWE.
- Secure against malicious aggregators by at most doubling communication cost per party.

## Some limitations of current HE-based solutions

**Non-Colluding Assumption:** Single-Key HE [3] imposes a non-colluding assumption between the aggregator and the owner of the secret key SK.



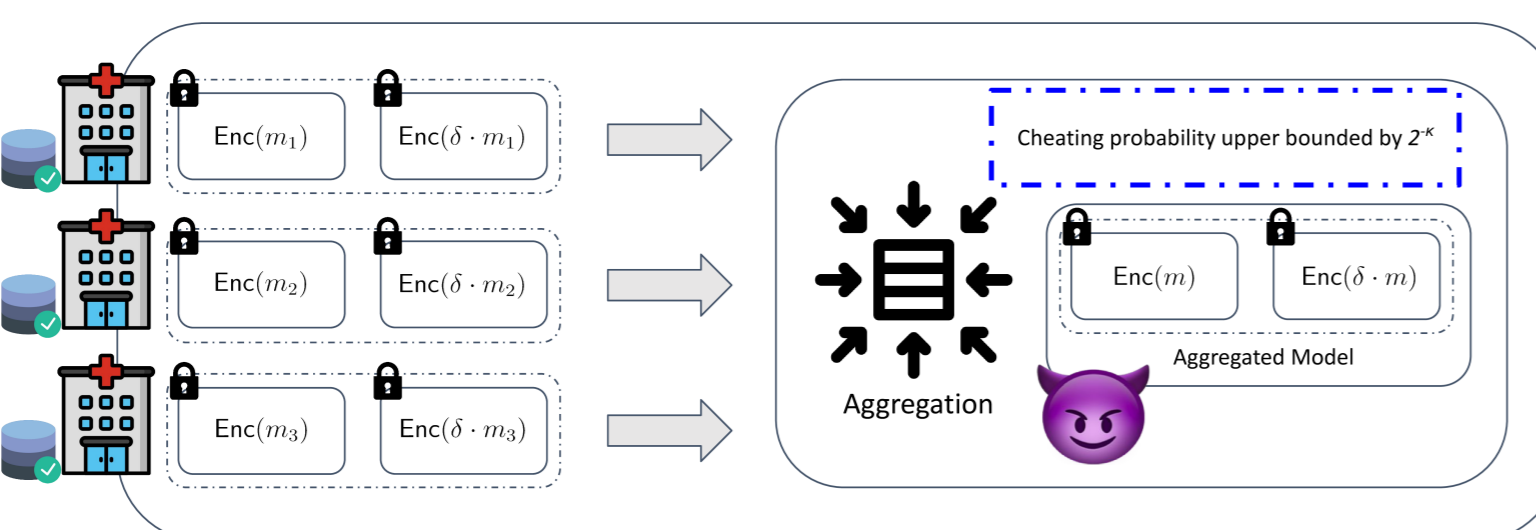
**Public Keys:** Both Single-Key HE [3] and Threshold HE [4] give access to encryptions of zero (i.e., PK = Enc(0)) under the global secret key SK.



**Dishonest Data Owners:** A dishonest Data Owner (DO) could easily generate a valid encryption of the global secret key by only having access to the PK.

## An upgrade to malicious aggregators

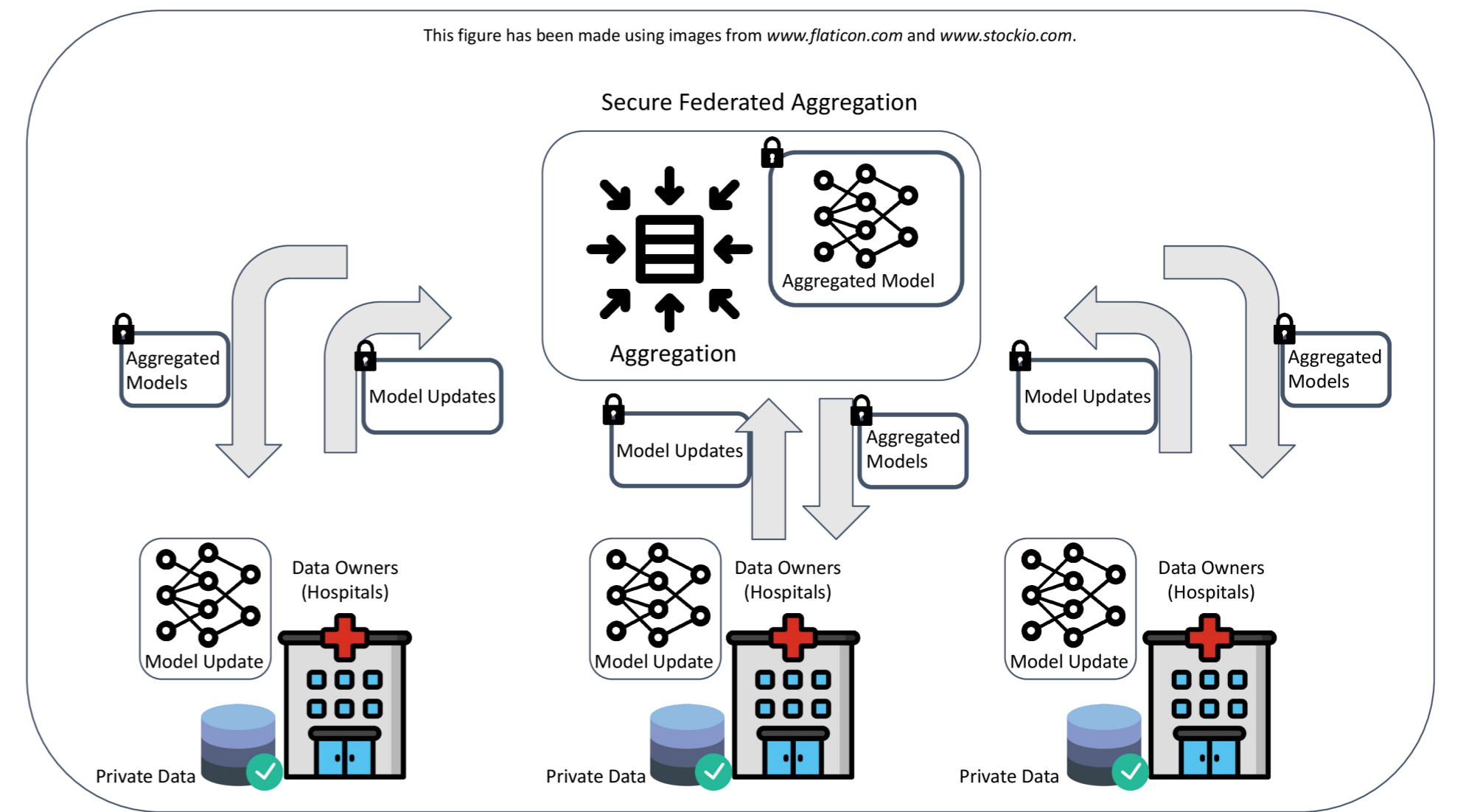
**Limiting ciphertexts' malleability:** By assuming the Common Reference String (CRS) model, a different "a" term is fixed among all Data Owners during each aggregation round.



- The Aggregator can only apply additive transformations without being detected.
- An extra condition check can be embedded into Secret-Key ciphertexts (e.g., δ · m with δ unknown to aggregator). This verifies the honest behavior during aggregation.

## Proposed HE-based Protocol

**High-level view:** Our HE-based protocol for secure aggregation. See [2] for more details.



**Protocol setup:**

- In the CRS model, DOs have access to a common uniformly random  $a$  per round.
- All DOs have access to one random polynomial share of zero:  $\text{share}_i = r^{(i)}$ .

**Workflow for a round of our secure aggregation protocol (semi-honest example):**

1. DOs encrypt their inputs: The  $i$ -th DO ( $\forall i$ ) encrypts its model update  $m_i$  as:

$$b_i = a(s_i + r^{(i)}) + e_i + q/p \cdot m_i.$$

2. Aggregation step:  $b = \sum_i b_i = a(s + \sum_i r^{(i)}) + e = a \cdot \underbrace{s}_{\sum_i s_i} + \underbrace{e}_{\sum_i e_i} + q/p \cdot \underbrace{m}_{\sum_i m_i}$ .

Finally, the aggregator sends back  $\text{share}^{(\text{agg})} = \lfloor b \rfloor_p$  to the DOs.

3. Distributed decryption:

- The  $i$ -th DO ( $\forall i$ ) computes  $\text{share}^{(i)} = \lfloor a s_i \rfloor_p$  and makes it available to all DOs.
- All DOs compute  $\lfloor \text{share}^{(\text{agg})} - \sum_i \text{share}^{(i)} \rfloor_p$ .

## Comparison with other solutions

Next table compares our work with a representative set of HE and MPC solutions.

M: Model Size N: Number of DOs n: lattice dimension M = constant · n	Ours [2]	[5]	[3]	[4]	[6]
<b>Agg. Comp. Cost</b>	$O(MN)$ add.	$O(MN)$ mult.	$O(MN)$ add.	$O(MN)$ add.	$O(MN^2)$
<b>DO Comp. Cost</b>	LWE: $O(Mn)$ mult. RLWE: $O(M \log M)$ mult.	$O(M)$ exp.	$O(M \log M)$ mult.	$O(M \log M)$ mult.	$O(MN + N^2)$
<b>Total Com. Cost</b>	$O(MN)$	$O(MN)$	$O(MN)$	$O(MN)$	$O(MN + N^2)$
<b>Multiple Keys</b>	✓	✗	✗	✓	✓
<b>Passive parties</b>	✓	✓	✓	✓	✓
<b>Malicious Agg.</b>	✓ Verify Agg.	✓ Verify Agg.	✗	✗	✓ only DOs input privacy if $T > N/2$
<b>Assumptions</b>	LWE/RLWE	Paillier	RLWE	RLWE	$T$ non-colluding DOs
<b>Flexible Dec.</b>	✓ only DOs contributing to aggregated model	✗	✗	✗	✓ required $T$ out of $N$ DOs

- **HE-based aggregation:** We include RLWE-based Single-Key [3] and Multi-Key [4] schemes. Also Paillier with verifiable computation for malicious aggregators [5].
- **MPC-based aggregation:** We include a work [6] relying on Shamir's Secret Sharing.

## References

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- [3] Arnaud Grivet Sébert, Renaud Sirdey, Oana Stan, and Cédric Gouy-Pailler, "Protecting data from all parties: Combining FHE and DP in federated learning," *CoRR*, vol. abs/2205.04330, 2022.
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